Finite-time singularities in the dynamics of hyperinflation in an economy

Martín A. Szybisz
Departamento de Economía, Facultad de Ciencias Económicas, Universidad de Buenos Aires, Av. Córdoba 2122, RA-1120 Buenos Aires, Argentina

Leszek Szybisz*
Laboratorio TANDAR, Departamento de Física, Comisión Nacional de Energía Atómica, Av. del Libertador 8250, RA-1429 Buenos Aires, Argentina;
Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria, RA-1428 Buenos Aires, Argentina;
and Consejo Nacional de Investigaciones Científicas y Técnicas, Av. Rivadavia 1917, RA-1033 Buenos Aires, Argentina
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The dynamics of hyperinflation episodes is studied by applying a theoretical approach based on collective "adaptive inflation expectations" with a positive nonlinear feedback proposed in the literature. In such a description it is assumed that the growth rate of the logarithmic price, $r(t)$, changes with a velocity obeying a power law which leads to a finite-time singularity at a critical time t_c. By revising that model we found that, indeed, there are two types of singular solutions for the logarithmic price, $p(t)$. One is given by the already reported form $p(t) \approx (t_c - t)^{-\alpha}$ (with $\alpha > 0$) and the other exhibits a logarithmic divergence, $p(t) \approx \ln[1/(t_c - t)]$. The singularity is a signature for an economic crash. In the present work we express $p(t)$ explicitly in terms of the parameters introduced throughout the formulation avoiding the use of any combination of them defined in the original paper. This procedure allows to examine simultaneously the time series of $r(t)$ and $p(t)$ performing a linked error analysis of the determined parameters. For the first time this approach is applied for analyzing the very extreme historical hyperinflations occurred in Greece (1941–1944) and Yugoslavia (1991–1994). The case of Greece is compatible with a logarithmic singularity. The study is completed with an analysis of the hyperinflation spiral currently experienced in Zimbabwe. According to our results, an economic crash in this country is predicted for these days. The robustness of the results to changes of the initial time of the series and the differences with a linear feedback are discussed.

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I. INTRODUCTION

Nowadays there is a significant interest in applications of physical methods in social and economical sciences [1–3]. In the field of economy, it has been found that the logarithmic change of the market price in the case of a hyperinflation episode shows some universal characteristics similar to those observed in physics. Mizuno, Takayasu, and Takayasu (MTT) [4] showed that in such a regime the price index increases more rapidly than a simple exponential law. Subsequently, it was suggested by Sornette, Takayasu, and Zhou (STZ) [5] that such a superexponential law indeed finishes with a finite-time singularity like that exhibited by some physical systems.

Let us recall that the rate of inflation $i(t)$ is defined as

$$i(t) = \frac{P(t) - P(t - \Delta t)}{P(t - \Delta t)} = \frac{P(t)}{P(t - \Delta t)} - 1, \qquad (1.1)$$

where $P(t)$ is the price at time t and Δt is the period of the measurement. In 1956, Cagan published *The Monetary Dynamics of Hyperinflation* [6], generally regarded as the first serious study of hyperinflation and its effects. He defined that "inflation rates per month exceeding 50%" determine a scenario of hyperinflation. However, no precise definition of hyperinflation is universally accepted, in informal usage the term is often applied to much lower rates. In economics, usually the terminology "hyperinflation" is applied in a rather rough sense to specify very high inflation that is "out of control," a condition in which prices increase rapidly as a currency loses its property as medium of exchange, store of value, and unit of account. In such a regime very important changes in relative prices occur and investment returns become unpredictable. These effects are accompanied with a strong devaluation of the currency and a decline in real public revenues increasing the fiscal deficit. Hyperinflation reduces real value of taxes collected, which are often set in nominal terms, and by the time they are paid, the real value has fallen. This feature is known as the Olivera-Tanzi effect, after Olivera [7] and Tanzi [8] who were the first to interpret it by means of standard analytical tools [9]. The occurrence of the Olivera-Tanzi effect may impulse a rapid expansion of nominal money and credit. For instance, Anušić and Švaljek [10] described the actions implemented in order to avoid the Olivera-Tanzi effect in several countries focusing a special attention to Croatia after the separation from Yugoslavia.

The scenario of a hyperinflation produces an important crisis in the population. Since the real investment, loans, growth, and development diminish, while the unemployment and political unrest significantly grow, a generalized instability is usually developed. Moreover, some of these effects may continue after the hyperinflation has been stopped. Therefore, models of hyperinflation are considered very use-

*Corresponding author; szybisz@tandar.cnea.gov.ar

ful by macroeconomists because detecting hyperinflation in an early stage might contribute to avoid such tragedy.

Since a long time ago it is known that in the case of moderate inflation the prices exhibit an exponential growth [6]. The superexponential growth of prices observed during a hyperinflation regime can be obtained from a nonlinear positive feedback process in which the past market price growth influences the people's expected future price, which itself impacts the *a posteriori* realized market price [4,5]. This process is fundamentally based on the mechanism of adaptive inflationary expectation, and it is similar to the positive feedbacks occurring during transmission of information due to imitative and herd behaviors [3,11]. In the present work, we report the first studies of the very extreme hyperinflations of Greece and Yugoslavia performed within this framework. The results strongly support the description based on the collective behavior outlined above. It is worthy of noticing that in Greece the inflation of the first ten days of November 1944 was $8.55\times10^7\%$ (see Fig. 3 in [6]) and in Yugoslavia the price level rose from 1 in December 1990 to the extraordinary order of 10^{50} in January 1994. The study is finished with the analysis of the hyperinflation developing just now in Zimbabwe. This case is very encouraging because the spiral of increasing prices is still not finished and a "genuine" prediction for the critical time can be done. Preliminary results of this work have already been posted elsewhere [12]. After that publication the economic situation in Zimbabwe became much worse.

In Sec. II we revise the theoretical formulations published in Refs. [4,5] introducing a careful treatment of the initial time of the series of data, t_0, and the period of measurements, Δt. It is shown that besides the singular solution reported in [5] the logarithmic price may also diverge logarithmically. The evolution of the price index is expressed explicitly in terms of the free parameters introduced along the theoretical formulation avoiding any combination of them. In this way one may analyze simultaneously the growth rate (GR) of logarithmic price and the price level itself. This procedure allows an examination of the self-consistency of the model. In Sec. III the results of the analyses are reported together with some reference to the economic and social situations. The uncertainties in the free parameters are also estimated. Finally, the main conclusions are summarized in Sec. IV.

II. THEORETICAL BACKGROUND

In the academic financial literature, the simplest and most robust way to account for inflation is to take logarithm. Hence, the continuous rate of change in prices is defined as

$$C(t) = \frac{\partial \ln P(t)}{\partial t}. \tag{2.1}$$

Usually the derivative of Eq. (2.1) is expressed in a discrete way as

$$C\left(t+\frac{\Delta t}{2}\right) = \frac{[\ln P(t+\Delta t) - \ln P(t)]}{\Delta t} = \frac{1}{\Delta t}\ln\left[\frac{P(t+\Delta t)}{P(t)}\right]. \tag{2.2}$$

The growth rate (GR) of logarithmic price over one period (a mean velocity of change) is defined as

$$\begin{aligned} r\left(t+\frac{\Delta t}{2}\right) &\equiv C\left(t+\frac{\Delta t}{2}\right)\Delta t \\ &= \ln\left[\frac{P(t+\Delta t)}{P(t)}\right] \\ &= \ln[1+i(t+\Delta t)] \\ &= p(t+\Delta t) - p(t), \end{aligned} \tag{2.3}$$

where a widely utilized definition $p(t)=\ln P(t)$ is introduced.

A. Cagan's model of inflation

In his pioneering work, Cagan [6] proposed a model of inflation based on the mechanism of adaptive inflationary expectation with positive feedback between realized growth of the market price $P(t)$ and the growth of people's averaged expectation price $P^*(t)$. These two prices are thought to evolve due to a positive feedback mechanism: an upward change of market price $P(t)$ in a unit time Δt induces a rise in the people's expectation price $P^*(t)$, and such an anticipation pushes on the market price. Cagan's assumptions may be cast into the following equations:

$$\frac{P(t+\Delta t)}{P(t)} = 1 + i(t+\Delta t) = \frac{P^*(t)}{P(t)} = \frac{P^*(t)}{P^*(t-\Delta t)}, \tag{2.4}$$

$$\frac{P^*(t+\Delta t)}{P^*(t)} = \frac{P(t)}{P(t-\Delta t)} = 1 + i(t). \tag{2.5}$$

Now, one may introduce

$$r^*\left(t+\frac{\Delta t}{2}\right) \equiv C^*\left(t+\frac{\Delta t}{2}\right)\Delta t = \ln\left[\frac{P^*(t+\Delta t)}{P^*(t)}\right]. \tag{2.6}$$

Equations (2.4) and (2.5) are equivalent to

$$r\left(t+\frac{\Delta t}{2}\right) = r^*\left(t-\frac{\Delta t}{2}\right), \tag{2.7}$$

$$r^*\left(t+\frac{\Delta t}{2}\right) = r\left(t-\frac{\Delta t}{2}\right), \tag{2.8}$$

whose solution is $r(t+\Delta t)=r(t-\Delta t)$, indicating a constant finite GR equal to its initial value

$$r(t) = r(t_0) = r_0. \tag{2.9}$$

The cumulated price index (CPI) is given by

$$P(t) = P(t_0)\exp\left[\int_{t_0}^{t} r(t')\frac{dt'}{\Delta t'}\right], \tag{2.10}$$

where to ensure a dimensionless exponent the time should be measured in the same units as the period $\Delta t'$ of the statistical data (i.e., days, months, or years). For $r(t)=r_0$ the integral of Eq. (2.10) leads to a steady state exponential inflation

$$P(t) = P_0 \exp\left[r_0\frac{(t-t_0)}{\Delta t}\right], \tag{2.11}$$

with $P_0=P(t_0)$. Here the role of Δt becomes clear, it fixes the time scale. Upon taking logarithms, this form may be written as a linear expression in t

$$p(t) = p_0 + r_0\left(\frac{t - t_0}{\Delta t}\right). \tag{2.12}$$

B. Model for hyperinflation episodes

The generalization of the Cagan's model for describing a regime of hyperinflation proposed by STZ [5] is based on a nonlinear version of the feedback process. These authors kept Eq. (2.7) and replaced Eq. (2.8) by

$$r^*\left(t + \frac{\Delta t}{2}\right) = r\left(t - \frac{\Delta t}{2}\right) + 2a_p\left[r\left(t - \frac{\Delta t}{2}\right)\right]^{1+\beta}, \quad \text{with } \beta > 0. \tag{2.13}$$

Here a_p is the dimensionless feedback's strength. Let us emphasize that the Cagan's formulation is retrieved by setting $a_p=0$, while that by fixing $\beta=0$ one gets the double-exponential growth of CPI derived by MTT [4]. Since β can be a noninteger real number this formalism requires $r(t) \geq 0$. It is reasonable to assume that this condition is statistically fulfilled in a hyperinflation regime. Even after periods of high inflation or hyperinflation the price level usually does not become smaller (no deflation with negative r), but instead the speed of growth diminishes [see, e.g., the table of the International Monetary Fund (IMF) [13]]. The case of Italy displayed in Fig. 3 of MTT [4] illustrates such an evolution.

The approach based on Eq. (2.13) provides a prediction of the future path until its end at a critical time t_c, while the model of MTT [4] does not predicts any final time. In practice, an hyperinflation regime is expected to finish before t_c because governments and central banks are forced to act avoiding the finite-time singularity. Such actions are the equivalent of finite-size and boundary condition effects in physical systems undergoing similar finite-time singularities. Hyperinflation regimes are of special interest because they emphasize in an almost pure way the impact of a collective behavior of people interacting via their expectations.

By introducing Eq. (2.13) into Eq. (2.7) one gets

$$r(t + \Delta t) = r(t - \Delta t) + 2a_p[r(t - \Delta t)]^{1+\beta}. \tag{2.14}$$

Taking the continuous limit of this expression one obtains the following equation for the time evolution of r:

$$\frac{dr}{dt} = \frac{a_p}{\Delta t}[r(t)]^{1+\beta} = a_1[r(t)]^{1+\beta}, \tag{2.15}$$

where a_1 is a positive coefficient with dimensions of the inverse of time used in STZ [5].

Let us show that for $\beta=0$ one recovers the MTT formulation. In this case one has

$$\frac{1}{r(t)}\frac{dr}{dt} = \frac{a_p}{\Delta t}, \tag{2.16}$$

hence, the GR becomes

$$r(t) = r_0 \exp\left[a_p \frac{(t - t_0)}{\Delta t}\right]. \tag{2.17}$$

In turn, after introducing this expression in Eq. (2.10) one arrives at a double exponential for the CPI

$$P(t) = P(t_0)\exp\left\{\frac{r_0}{a_p}\left(\exp\left[a_p\frac{(t - t_0)}{\Delta t}\right] - 1\right)\right\}. \tag{2.18}$$

Once the hyperinflation is started, the term -1 becomes negligibly small compared to the exponential, under such circumstances this expression coincides with Eqs. (2) and (3) of MTT [4]. It is important to notice that the discrete version of the MTT model also leads to a double exponential for the CPI. Of course, the logarithmic of CPI as a function of t follows basically an exponential law

$$p(t) = p_0 + \frac{r_0}{a_p}\left\{\exp\left[a_p\frac{(t - t_0)}{\Delta t}\right] - 1\right\}. \tag{2.19}$$

For $(t-t_0) \ll \Delta t/a_p$ this expression leads to the Cagan's result given by Eq. (2.12).

For $\beta > 0$ the GR itself also follows a power law exhibiting a singularity in finite-time

$$r(t) = r_0\left[\frac{1}{1 - \beta a_p r_0^{\beta}\left(\frac{t - t_0}{\Delta t}\right)}\right]^{1/\beta} = r_0\left(\frac{t_c - t_0}{t_c - t}\right)^{1/\beta}. \tag{2.20}$$

The critical time t_c is determined by the initial GR r_0, the exponent β, and the strength parameter a_p

$$t_c - t_0 = \frac{1}{\beta a_1 r_0^{\beta}} = \frac{\Delta t}{\beta a_p r_0^{\beta}}. \tag{2.21}$$

In the discrete version of this nonlinear model, $r(t)$ follows a double-exponential law, while $P(t)$ increases obeying a triple-exponential law.

In the continuous version the CPI is obtained by integrating the expression for $r(t)$ given by Eq. (2.20) according to Eq. (2.10). For $\beta \neq 1$ it becomes

$$\ln\left[\frac{P(t)}{P_0}\right] = \int_{t_0}^{t} r(t')\frac{dt'}{\Delta t'} = \frac{r_0^{1-\beta}}{(1-\beta)a_p}\left\{\left[\frac{1}{1 - \beta a_p r_0^{\beta}\left(\frac{t - t_0}{\Delta t}\right)}\right]^{(1-\beta)/\beta} - 1\right\}, \tag{2.22}$$

while for $\beta=1$ it reads

$$\ln\left[\frac{P(t)}{P_0}\right] = \frac{1}{a_p}\ln\left[\frac{1}{1 - a_p r_0\left(\frac{t - t_0}{\Delta t}\right)}\right]. \tag{2.23}$$

The latter solution was not considered in STZ [5]. In both cases, in the limit $t-t_0 \ll t_c-t_0$ one obtains $\ln[P(t)/P_0] \simeq r_0(t-t_0)/\Delta t$ recovering the linear expression Eq. (2.12) of the Cagan model.

After introducing t_c in the expressions given above, the time dependence of $p(t)$ exhibits the following three different regimes depending on the size of β:

(i) For $0<\beta<1$ one gets a finite-time singularity in the logarithm of CPI according to the power law

$$p(t) = p_0 + \frac{\beta r_0}{1-\beta}\left(\frac{t_c - t_0}{\Delta t}\right)\left[\left(\frac{t_c - t_0}{t_c - t}\right)^{(1-\beta)/\beta} - 1\right]. \tag{2.24}$$

(ii) For $\beta=1$ the logarithmic CPI exhibits a logarithmic divergence

$$p(t) = p_0 + r_0\left(\frac{t_c - t_0}{\Delta t}\right)\ln\left(\frac{t_c - t_0}{t_c - t}\right). \tag{2.25}$$

In both these regimes (i) and (ii) the price exhibits a finite-time singularity at the same critical value t_c as GR. Hence, these solutions correspond to a genuine divergence of $\ln P(t)$.

(iii) For $\beta>1$ one gets a finite-time singularity in $r(t)$ but the logarithmic CPI evolve as

$$p(t) = p_0 + \frac{\beta r_0}{\beta - 1}\left(\frac{t_c - t_0}{\Delta t}\right)\left[1 - \left(\frac{t_c - t}{t_c - t_0}\right)^{(\beta-1)/\beta}\right]. \tag{2.26}$$

As time approaches the critical value t_c the logarithmic CPI converges to the value

$$p(t \to t_c) = p_0 + \frac{\beta r_0}{\beta - 1}\left(\frac{t_c - t_0}{\Delta t}\right), \tag{2.27}$$

leading to equilibrium, this limit is reached more rapidly when $(\beta-1)/\beta \to 1$.

In order to make a direct contact with the formulas utilized for the analysis in STZ [5], one may introduce the change in variable,

$$\alpha = \frac{1-\beta}{\beta}, \tag{2.28}$$

and arrive at

$$p(t) = p_0 + \frac{r_0}{\alpha}\left(\frac{t_c - t_0}{\Delta t}\right)\left[\left(\frac{t_c - t_0}{t_c - t}\right)^{\alpha} - 1\right]. \tag{2.29}$$

In turn, this expression may be cast into the form of Eq. (15) in [5]

$$p(t) = A + B(t_c - t)^{-\alpha}, \tag{2.30}$$

with

$$A = p_0 - \frac{r_0}{\alpha}\left(\frac{t_c - t_0}{\Delta t}\right) \tag{2.31}$$

and

$$B = \frac{r_0}{\alpha \Delta t}(t_c - t_0)^{1+\alpha}. \tag{2.32}$$

Since

$$\beta = \frac{1}{1+\alpha}, \tag{2.33}$$

Eq. (2.20) takes the form

$$r(t) = r_0\left(\frac{t_c - t_0}{t_c - t}\right)^{1+\alpha}. \tag{2.34}$$

We must notice that in the expression given by STZ [5] there are misprints: (i) the coefficient a_1 should be dropped from their Eq. (14) and (ii) the form for t_c given just below that equation also presents typographical errors.

In the forms proposed for Eqs. (2.24) and (2.25) all the free parameters have their own physical meaning: t_c is the hyperinflation's end-point time, β is the exponent of the power law, r_0 is the initial growth of logarithmic price, and p_0 is the initial logarithmic price. While A and B are combinations of that parameters. Moreover, in the present formulation Eq. (2.20) for $r(t)$ and Eqs. (2.24) and (2.25) for $p(t)$ are written in terms of the same parameters allowing a simultaneous analysis of both these observables.

The types of singularities obtained for $r(t)$ and $p(t)$ are present in several physical systems. Let us mention here some very different examples. The nature of the finite-time singularity when a circular disk (Euler's disk) is spun upon a table and ultimately it comes to rest quite abruptly was discussed by Moffatt [14]. Bhattacharjee and Wang [15] found a finite-time vortex singularity in a model of three-dimensional Euler flows. In the case of classical fluids the thickness of the liquid-vapor interface on the coexistence curve shows a finite-temperature singularity when temperature T approaches the critical point T_c (see, e.g., Fig. 7 in [16]). In particular, the logarithmic singularity is exhibited by the specific heat of superfluid ^{4}He when T approaches the λ point $T_\lambda = 2.17$ K from below [see, e.g., Eq. (1) in [17]].

III. ANALYSIS AND NUMERICAL RESULTS

In order to get an estimation of t_c we fitted the measured data to expressions derived in the previous section. Unfortunately, the statistical files provided by the governments never provide an estimation of the error of measured inflations. However, in any measurement there is an intrinsic uncertainty which causes an error in the fitted parameters. Due to the lack of information on data's error, we were forced to get standard deviations directly for the fitting procedure. The parameters were determined by means of a nonlinear least-squares fit which minimizes the χ^2. As in STZ [5], no weighting of data was considered in the evaluation of χ^2. Hence, this quantity is directly the residue of the mean-square fit. The numerical task was accomplished by using a routine of the book by Bevington [18] cited as the first reference in chapters 15.4 and 15.5 of the more recent *Numerical Recipes* [19]. In this case the uncertainty in each parameter is directly obtained from the minimization procedure.

The measured CPI was fitted by doing two different assumptions for p_0. In one of them this parameter was fixed at the normalization value p_0 and in the other it was left free. In this way two sets of the parameters t_c, β, and r_0 were determined, allowing useful comparisons.

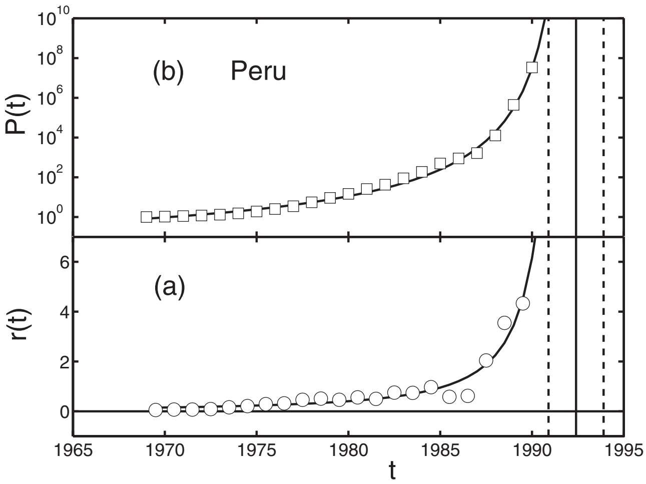

FIG. 1. (a) Yearly GR of the logarithmic price in Peru from 1969 to 1990 (circles). (b) Semilogarithmic plot of the yearly CPI during the same period normalized to $[P(t_0\!=\!1969)\!=\!1]$ (squares). The solid curves are four-parameter GR+CPI fits to Eqs. (2.20) and (2.24). In both panels the vertical solid line indicates the predicted critical time t_c and vertical dashed lines are the error quotes for it, see text for explanations.

First, we completed in the scope of the present framework the analysis of one of the cases studied in [5]. For this purpose, the episode of Peru was selected. Next, the results of our study of hyperinflation in Greece, Yugoslavia, and Zimbabwe are reported chronologically. The robustness of the determined parameters to changes in the initial time of the series was analyzed. This issue is discussed for the episodes occurred in Yugoslavia and Zimbabwe.

Since in the discrete versions of the MTT and STZ models the time dependence of the CPI follows a double- and a triple-exponential law, respectively, it is expected that differences should be mainly observed in the final stage of a hyperinflation. These models are compared in the case of Zimbabwe.

A. Hyperinflation in Peru

Figure 1 shows the yearly data of Peru measured during the period 1969–1990. The $r(t)$ is displayed in panel (a), while the $P(t)$ normalized to $P(t_0\!=\!1969)\!=\!1$ is plotted in a semilogarithmic scale in panel (b). First of all we fitted the CPI to Eq. (2.24) in order to check the procedure by comparing the results with those of STZ [5]. The obtained parameters t_c, β, r_0, p_0, its uncertainties, and the reduced χ are quoted in Table I. To allow a direct quantitative comparison the parameters α, A, and B evaluated with Eqs. (2.28), (2.31), and (2.32), respectively, are listed in Table II. The agreement between both series of parameters is perfect. The relatively large error in α mainly determines the size of the uncertainties in A.

In order to examine the self-consistency of the model we fitted $r(t)$ to Eq. (2.20). The obtained parameters listed in Table I are consistent within the quoted uncertainties with that yielded by the CPI fit. Next, we fitted simultaneously $r(t)$ to Eq. (2.20) and $\ln P(t)$ to Eq. (2.24), denoting this fit as GR+CPI. The obtained parameters with the corresponding uncertainties are included in Table I together with the χ. The values of the parameters t_c, β, and r_0 are consistent within the error bars. These results support the self-consistency of the model. The quality of the fit can be observed in Fig. 1, panel (b) of this plot may be compared with Fig. 2 of STZ [5].

B. Greek catastrophic episode

Let us begin the description of the Greek hyperinflation by citing a fragment of the talk addressed by Garganas, Governor of the Bank of Greece. He said [23]: "in April 1941, the Axis powers occupied Greece. For several years, London became the seat of both the exiled Greek government and the Bank of Greece, with the bank's gold secretly transferred to South Africa. Within occupied Greece, the economic situation became increasingly grim and hundreds of thousands of Greeks died of hunger. The Axis powers forced the country to pay not only for the upkeep of the occupying troops but also for their military operations in Southeastern Europe. The puppet regime established by the occupiers forced the Bank of Greece to resort to the printing press. As a result, the country was beset with hyperinflation; between April 1941 and October 1944, the cost of living rose 2.3×10^9 times. In these difficult circumstances, the country's economic system collapsed. To give another example of the magnitude of inflation during the occupation, let us mention that in November 1944, immediately after liberation, a so-called "new" drachma was introduced; it was set equal to 50×10^9 "old" drachmas.

Figure 2 shows in panel (a) the GR of logarithmic price in Greece taken from Table B6 of [6] and in panel (b) a semilogarithmic plot of the CPI evaluated at the end of each month and normalized to $P(t_0\!=\!1941\!:\!04\!:\!30)\!=\!1$ (henceforth the notation Year:Month:Day will be used). The open stars are the values at 1944:11:10, i.e., it corresponds to the first ten days of November. In Fig. 2(b) one may observe the value 2.3×10^9 mentioned in the Garganas' talk [23]. A simple inspection of this figure indicates two well-differentiated regimes of inflation. This behavior can be understood in terms of different phases of the foreign occupation.

Over the period 1941:04 to 1942:10, the cumulated inflation has been about 130%. Hence, the level of a hyperinflation was still not reached and the CPI can be well described by the original theory of Cagan given by Eq. (2.12) with $p_0\!=\!0$. The value of r_0 and χ are quoted in Table I. The quality of the fit is quite good.

During winter 1942 and 1943 some initial success of the guerrilla diminished the tension in the population causing a period of small deflation over four months. This fact can be clearly seen in Fig. 2. However, the pressure of German elite troops was immediately increased and the Greek treasury was forced to pay huge amounts of "occupation expenses." Since the government of Greece could not meet such an obligation from fiscal taxation (Olivera-Tanzi effect [7–10]) new money was printed (seigniorage). Due to the general scenario of the war, the Germans were forced to evacuate

TABLE I. Analyzed hyperinflations, critical time t_c, and remaining parameters with the estimated uncertainties and χ.

Country	Currency	Period	Parameters				Fitted	
			t_c	β	r_0	p_0	Observables	χ
Peru[a]	Inti	1969–1990	1991.29 ± 0.74	0.77 ± 0.13	0.180 ± 0.027	-0.37 ± 0.26	CPI	0.322
			1993.4 ± 3.0	0.50 ± 0.24	0.112 ± 0.079		GR	0.302
			1992.4 ± 1.5	0.61 ± 0.14	0.141 ± 0.025	-0.19 ± 0.21	GR+CPI	0.330
			1992.3 ± 1.6	0.61 ± 0.16	0.141 ± 0.027	0	GR+CPI	0.322
Greece[b]	Drachma	1941:04–1942:10			0.271 ± 0.094	0	GR	0.010
		1943:02–1944:10	$1944:12:02\pm18$	0.86 ± 0.14	0.211 ± 0.028	3.91 ± 0.23	CPI	0.230
			$1944:11:16\pm07$	1.01 ± 0.27	0.243 ± 0.134		GR	0.250
			$1944:11:17\pm04$	0.96 ± 0.09	0.220 ± 0.025	3.98 ± 0.21	GR+CPI	0.254
			$1944:11:18\pm04$	0.93 ± 0.09	0.208 ± 0.021	4.17	GR+CPI	0.278
			$1944:11:16\pm04$	1	0.243 ± 0.072	4.09 ± 0.32	GR+CPI	0.251
			$1944:11:16\pm03$	1	0.238 ± 0.051	4.17	GR+CPI	0.244
Yugoslavia[c]	Dinar	1990:12–1994:01	$1994:03:10\pm04$	0.65 ± 0.02	0.332 ± 0.018	-1.52 ± 0.30	CPI	0.930
			$1994:02:28\pm07$	0.71 ± 0.06	0.374 ± 0.010		GR	0.607
			$1994:03:15\pm04$	0.61 ± 0.02	0.288 ± 0.008	0	GR+CPI	1.019
			$1994:03:07\pm03$	0.66 ± 0.02	0.330 ± 0.010	-1.22 ± 0.15	GR+CPI	0.842
Zimbabwe[d]	ZW dollar	1979:12–2007:12[e]	$2009:08\pm13$	0.62 ± 0.13	0.084 ± 0.023	0.06 ± 0.40	CPI	0.248
			$2008:06\pm13$	0.86 ± 0.44	0.130 ± 0.125		GR	0.244
			$2009:03\pm08$	0.66 ± 0.10	0.090 ± 0.016	0.03 ± 0.17	GR+CPI	0.255
			$2009:03\pm08$	0.67 ± 0.10	0.091 ± 0.014	0	GR+CPI	0.254
		1998:12–2008:07[f]	$2008:09:06\pm22$	0.89 ± 0.05	0.047 ± 0.003	-0.10 ± 0.09	GR+CPI	0.202
			$2008:09:12\pm23$	0.87 ± 0.05	0.045 ± 0.002	0	GR+CPI	0.211

[a]Inflation data are taken from a Table published by the IMF [13].
[b]Inflation data are taken from Ref. [6].
[c]Inflation data are taken from Refs. [20,21].
[d]Inflation data are taken from Ref. [22].
[e]Yearly cumulated data.
[f]Monthly data.

mainland Greece in October 1944, finishing their withdrew on November 2. Then the exiled government returned to Athens and two resistance groups (left and right orientated organizations) immediately began to fight for power leading to a disastrous civil war. The very difficult situation during the last years of the occupation caused the catastrophic hyperinflation displayed in Fig. 2. Any attempt to control prices failed and the fall off of production in devastated Greece's economy led to the collapse of the normal markets and to an increase in the black market. After liberation, in the first ten days of November 1944 the inflation rose the incredible value $8.55\times10^{7}\%$.

TABLE II. The exponent α and the combinations of parameters A and B determined from the fit of the price level of Peru.

α	Parameters		Ref.
	A	B	
0.3	−14.17	34.0	[5]
0.29 ± 0.22	$-14.\pm11$	$34.\pm6$	PW[a]

[a]PW stands for present work.

In a first step, the monthly values of the CPI from 1942:02 to 1944:10 and data of $r(t)$, including in this case the value $r(t{=}1944:11:10)$, were separately fitted to Eqs. (2.24) and (2.20), respectively. The adjusted free parameters are listed in Table I. Notice that the result for β is consistent with unity. Next, we performed simultaneous GR+CPI fits of $r(t)$ and $p(t)$ to the same equations. Two different fits were done, one setting $p_0{=}p(t{=}1942:02){=}4.17$ and other considering p_0 as a free parameter. Both sets of determined parameters are also quoted in Table I, in both cases $\beta\simeq1$. Therefore, in a final step we assumed $\beta{=}1$ and performed two- and three-parameter GR+CPI fits to Eqs. (2.20) and (2.25). The results are included in Table I. Figure 2 shows that the three-parameter fit reproduces quite well the measured data indicating that the observables are fully consistent with a logarithmic finite-time singularity. The obtained critical time predicts the definitive crash would be on 1944:11:16 that is a few days after November 10. Moreover, it is important to emphasize that even the two-parameter fit ($p_0{=}4.17$) describes very well the evolution of the observables.

The stratification of wealth caused by hyperinflation and black markets during the occupation seriously hindered postwar economic development. The Greek government under-

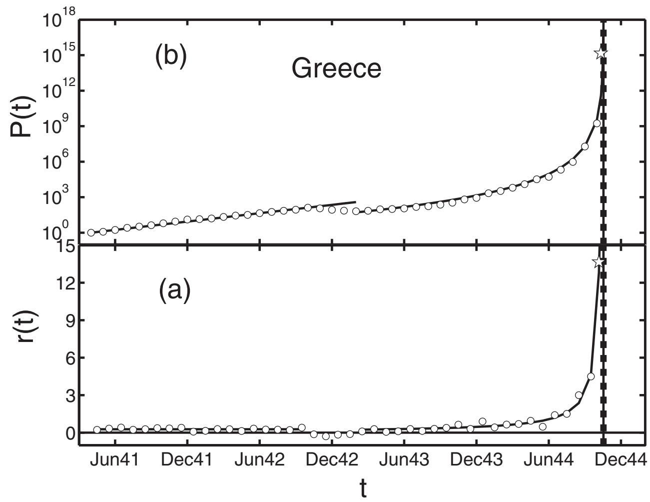

FIG. 2. (a) Monthly GR of the log-price in Greece from 1941:04 to 1944:10 (circles). (b) Semilogarithmic plot of the monthly CPI in the same period normalized to $P(t_0=1941:04)=1$ (squares). In both panels the open star is the value of 1944:11:10. The horizontal line in (a) is the average growth rate in the period 1941:04–1942:10, while the straight line in (b) is given by Eq. (2.12) in the same period. The solid curves in (a) and (b) are four-parameter GR +CPI fits of data of the period 1943:02–1944:10 to Eqs. (2.20) and (2.25) (see text). The vertical solid line is the predicted critical time t_c, while the vertical dashed lines indicate its error bars.

took several stabilization efforts spread over a couple of years before price level stability was achieved. These facts are described in the books written by Palairet [24] and Lykogiannis [25]. The efforts to confront the hyperinflation consisted of a currency conversion (convertibility of the new drachma into British Military Authority Pounds), the creation of an independent supracentral bank limiting the government's overdraft at the Bank of Greece, and a few fiscal reforms to increase taxes or reduce expenditures.

C. Yugoslavia: The worst episode in History

The residual Yugoslavia has experienced the highest recorded hyperinflation in history. This episode occurred during a period of two difficult circumstances: (i) the transformation from central planning to a rather free market economy, and (ii) the disastrous civil war 1991–1994. The disintegration of the former Yugoslavia led to decreased output and fiscal revenue, while transfers to the Serbian population in Croatia and Bosnia-Herzegovina as well as military expenditure added to the fiscal problems. High inflation started to build up in 1991. In order to finance the increasing deficit the government printed money. However, there are studies suggesting that the Yugoslavian hyperinflation was not generated by excess expansion of money stock, but printed money was accommodated in this period [26].

The sources for the data of the complete cycle are documented in Petrović and Mladenović [20] and Nielsen [21]. Figure 3(a) shows the time series of the GR of logarithmic price in the period 1990:12 to 1994:01. The evaluated monthly CPI is plotted in Fig. 3(b). The monthly inflation

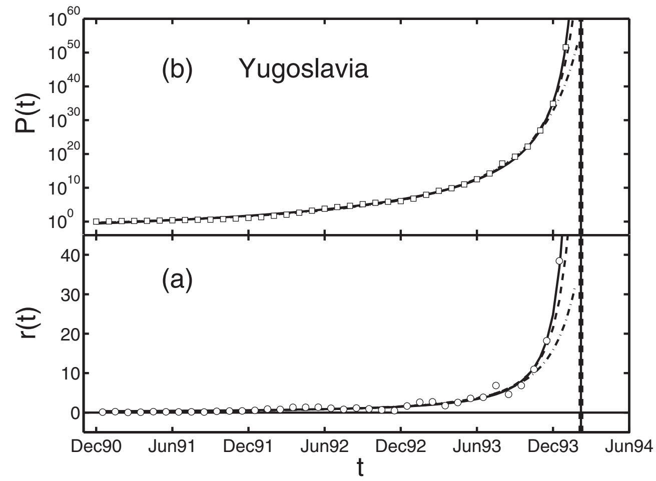

FIG. 3. (a) Monthly GR of the logarithmic price during the hyperinflation of Yugoslavia from 1990:12 to 1994:01 (circles). (b) Semilogarithmic plot of the monthly CPI normalized to $P(t_0=1990:12)=1$ (squares). The solid curves in (a) and (b) are four-parameter GR+CPI fits of all the data to Eqs. (2.20) and (2.24). In both panels, the solid vertical line is the corresponding predicted critical time t_c and the dashed vertical lines are its error bars. The dashed and dash-dotted curves are fits of data of the periods 1990:12–1993:12 and 1990:12–1993:11, respectively (see text).

had already risen the category of hyperinflation at the beginning of 1992 and accelerated further despite the price freeze attempted in the end of 1993:08. The overall impact of hyperinflation on the price index reached about 10^{50}, this feature is also shown in Fig. 1 of [21]. Through this period several subsequent currency reforms were introduced. At the end of 1993 the highest denomination was 500 000 000 000 dinars. This is the bill with largest number of zeros ever printed [27]. Many monetary transactions were done in west German marks rather than in local currency.

This hyperinflation has been studied in a number of papers by using methods developed in the framework of economics. For instance, we can mention the papers of Petrović and Mladenović [20] and Nielsen [21] already cited above. These authors analyzed data up to 1993:11 only, considering the CPI for 1993:12 and 1994:01 as unreliable. This procedure is in line with standard studies of hyperinflation, where the last few observations are often ignored due to the fact that a very big increase in the CPI does not match the systematics because such models do not contain structural information over the divergence at the end of a hyperinflation.

As in the case of Greece, in a first step, all the CPI and GR values displayed in Fig. 3 were separately fitted to Eqs. (2.24) and (2.20), respectively. The results are quoted in Table I. The value of χ indicates that the quality of the fit is good. Next, we performed two simultaneous GR+CPI fits of $r(t)$ and $p(t)$, in one of them setting $p_0=0$ and in the other considering p_0 as a free parameter. The results are also included in Table I. There is a satisfactory agreement between the four sets of parameters. The solid curves displayed in Fig. 3 were evaluated with t_c, β, r_0, and p_0 obtained from the four-parameter GR+CPI fit. The obtained t_c predicts a crash at the beginning of 1994:03. The hyperinflation was stopped

TABLE III. Comparison of critical time t_c, β, r_0, and p_0 determined from four-parameter GR+CPI fits in the cases of monthly data of residual Yugoslavia and Zimbabwe when: (i) the latest measured values are dropped and (ii) for different initial times.

			Parameters				
Country	Period	Final CPI	t_c	β	r_0	p_0	χ
Yugoslavia	1990:12–1993:11	0.9×10^{27}	$1994:07:07\pm33$	0.46 ± 0.04	0.257 ± 0.014	-0.73 ± 0.16	0.702
	1990:12–1993:12	0.7×10^{35}	$1994:04:11\pm12$	0.57 ± 0.03	0.293 ± 0.013	-0.92 ± 0.16	0.768
	1990:12–1994:01	0.4×10^{52}	$1994:03:07\pm03$	0.66 ± 0.02	0.330 ± 0.010	-1.22 ± 0.15	0.842
	1991:06–1994:01	0.2×10^{52}	$1994:03:01\pm03$	0.69 ± 0.02	0.469 ± 0.013	-0.60 ± 0.17	0.770
	1991:12–1994:01	0.3×10^{51}	$1994:02:27\pm03$	0.73 ± 0.02	0.679 ± 0.020	0.47 ± 0.18	0.796
	1992:06–1994:01	0.6×10^{48}	$1994:03:03\pm04$	0.68 ± 0.03	0.884 ± 0.031	-0.05 ± 0.21	0.903
Zimbabwe	1998:12–2008:05	0.5×10^{10}	$2008:09:05\pm59$	0.89 ± 0.08	0.047 ± 0.003	-0.10 ± 0.09	0.204
	1998:12–2008:06	0.5×10^{11}	$2008:09:08\pm41$	0.89 ± 0.07	0.047 ± 0.003	-0.10 ± 0.09	0.203
	1998:12–2008:07	0.1×10^{13}	$2008:09:06\pm22$	0.89 ± 0.05	0.047 ± 0.003	-0.10 ± 0.09	0.202
	1999:06–2008:07	0.1×10^{13}	$2008:08:27\pm20$	0.91 ± 0.05	0.051 ± 0.003	-0.11 ± 0.09	0.202
	1999:12–2008:07	0.8×10^{12}	$2008:08:27\pm22$	0.93 ± 0.06	0.055 ± 0.003	-0.07 ± 0.09	0.202
	2000:06–2008:07	0.6×10^{12}	$2008:08:23\pm21$	0.94 ± 0.06	0.060 ± 0.004	-0.05 ± 0.09	0.206

with a successful complete economic and currency reform at the end of 1994:01.

A further analysis of this hyperinflation was performed by doing four-parameter GR+CPI fits without including the latest data. In one case the analyzed period was 1990:12–1993:12 and in the other 1990:12–1993:11. The results are listed in Table III, where in order to facilitate the comparison the results of the full fit are also included. In each case the CPI at the end of the period is given. The changes are also displayed in Fig. 3. When less data are considered the influence of the failed attempt of freezing prices done at the end of 1993:08 causes a delay in t_c. This behavior may be clearly observed in both panels of Fig. 3. However, in spite of the changes a soon crash is always predicted.

In order to study the extent to which the determined parameters are robust to changes in the initial time of the series of GR and CPI we performed several additional fits. The results for $t_0=1991:06$, 1991:12, and 1992:06 (i.e., dropping 6, 12, and 18 months from the series examined initially) are reported in Table III together with the parameters for $t_0=1990:12$. A glance at this table indicates that the critical time t_c is stable within the uncertainties. The value of r_0 increases because the initial average ratio $P(t_0+\Delta t)/P(t_0)$ is bigger when t_0 is closer to t_c and the slight changes in β account for larger average curvatures of the included data.

D. Tragic current case of Zimbabwe

When the country achieved independence on 17 April 1980 under the name Zimbabwe (previously is was known as Southern Rhodesia or simply Rhodesia) the Zimbabwe dollar (ZW dollar, the official symbol is ZWD) was equivalent to 1.50 US dollar. Since the beginning there was a moderate but persistent structural inflation. Figure 4 shows the annual $r(t)$ and $P(t)$ taken from files of the Central Statistical Office (CSO) and the Reserve Bank of Zimbabwe (RBZ) [22]. By looking at this plot one may realize that an important acceleration of the CPI started at the end of the past century. This behavior appeared after the Zimbabwean government proceeded to finance among others (see, Makochekanwa [28]): (i) the expenditure to pay the war veteran's gratitude in 1997, (ii) the intervention in the Democratic Republic of Congo's war in 1998, and (iii) the expenses of a program of land reforms based on the redistribution of properties in 2000. As a matter of fact the latter undertaking led to a weakling of the agricultural industry and this in turn produced a fall of export revenues. The resulting deficit was covered by seigniorage. In this way Zimbabwe started to experience a real hyperinflation and the Olivera-Tanzi effect appeared.

The high inflation have caused a severe devaluation of currency and many organizations favor the use of US dollar instead of ZW dollar. The RBZ has already confirmed that

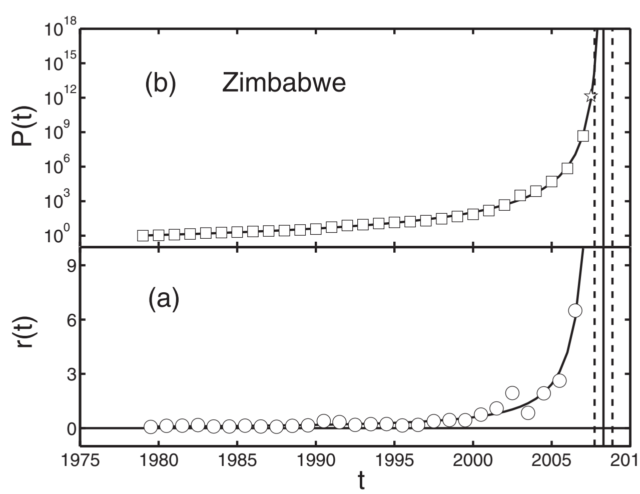

FIG. 4. (a) Yearly GR of the log-price in Zimbabwe from 1979:12 to 2007:12 (circles). (b) Semilogarithmic plot of the corresponding CPI normalized to $P(t_0=1979:12)=1$ (squares). The star is the CPI at end of 2008:06. The solid curves in (a) and (b) are four-parameter GR+CPI fits of the data to Eqs. (2.20) and (2.24). In both panels, the vertical solid line is the predicted critical time t_c and the dashed lines are its error bars.

certain farmers will receive foreign currency for their crops. In actual fact, Zimbabwe is closely tracking Germany's Weimar Republic in the early 1920s. There was a close parallelism between the evolution of exchange rate of the ZW dollar in the period 2005–2007 and that of German Marks in 1921–1923. At the beginning of 2008 the exchange rate in the back market was 1 US dollar $\simeq 3\times 10^6$ ZWD. By adding the three zeros slashed in 2006 one gets that the price of one US dollar increased since 1980 about 10^9, which coincides with the CPI shown in Fig. 4 at 2007:12.

In a preliminary report of this work [12], we analyzed annual data of CPI spanned from the foundation of the country up to 2007:12. The values are normalized to $P(t_0=1979:12)=1$. Such a study clearly showed that Zimbabwe was suffering the consequences of a serious episode of hyperinflation. It was predicted a crash within two years. The calculated CPI for the end of 2008 was about 10^{13} corresponding to an annual inflation of $5\times 10^6\%$. In spite of that situation, no important corrections in the economy were done.

After that report the inflations for 2007:11 and 2007:12 were corrected upwards and new data were released. Let us now report the analysis of the yearly $r(t)$ and $P(t)$ displayed in Fig. 4 including these corrections. As in the cases described above four different fits to Eqs. (2.20) and (2.24) were performed. The obtained values of the free parameters and χ are quoted in Table I, the agreement between the four sets of parameters is good. The adjustment yielded by the four-parameter GR+CPI fit is depicted in Fig. 4, where we added the CPI measured at 2008:06 which lies on the plotted curve. The predicted critical time t_c centered on 2009:03 predicts that the *blow up* of the economy will occur rather soon.

The rise of the CPI has sharply accelerated in 2008. The CSO stopped providing data on inflation in 2008:07. The consumer council of Zimbabwe and other observers questioned whether the figures provided officially reflected the true cost of living. They stated that real figure is almost certainly much larger. In any case, the Zimbabwe's inflation is already the highest in the world and has overpassed that of Latin America's in the 1980s. For instance, compare with the data of Peru plotted in Fig. 1.

Due to the importance of this case, we performed an additional analysis considering only the data influenced by the actions implemented by the government at the end of the last century. Fortunately, the RBZ [22] provides monthly measurements for this period. The data of $r(t)$ and $P(t)$ from 1998:12 to 2008:07 are plotted in Fig. 5. We performed three- and four-parameter GR+CPI fits of these values. The results are listed in Table I. The obtained t_c indicates that the economic crash would already happened during the last quarter of 2008. Therefore, to test the stability of the solution we also fitted data without including the measurements of 2008:07 and 2008:06, the results are listed in Table III. A glance at this table indicates that t_c remains almost fixed, only its error is enlarged when less data are considered.

The robustness of the determined parameters to changes of the initial time of the series of GR and CPI was examined by performing fits of the monthly data for $t_0=1999:06$, 1999:12, and 2000:06 (i.e., dropping 6, 12, and 18 months as in the case of Yugoslavia). The results are listed in Table III together with the parameters for $t_0=1998:12$. The conclusions are similar to that previously found for Yugoslavia. The critical time t_c is stable within the determined uncertainties, and the values of r_0 and β increase as expected.

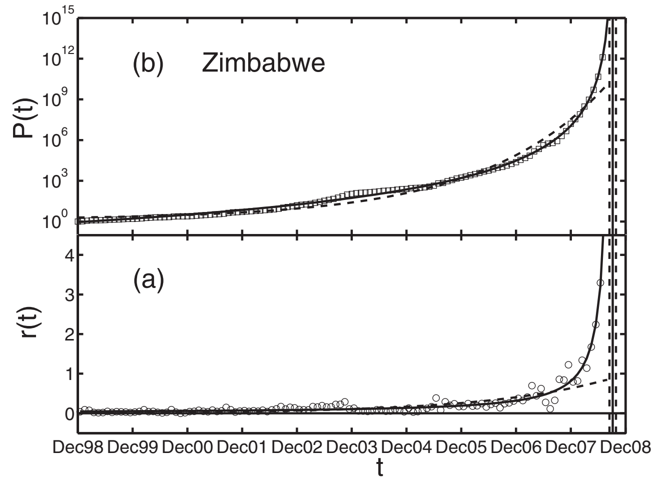

FIG. 5. (a) Monthly GR of the logarithmic price in Zimbabwe from 1998:12 to 2008:07 (circles). (b) Semilogarithmic plot of the corresponding CPI normalized to $P(t_0=1998:12)=1$ (squares). The solid curves in panels (a) and (b) are the four-parameter GR+CPI fits of the data to Eqs. (2.20) and (2.24), respectively, while the dashed curves in that panels are three-parameter fits to Eqs. (2.17) and (2.19). In both panels, the vertical solid line is the predicted critical time t_c and the dashed lines are its error bars.

Finally, we compare the two alternative descriptions proposed by the MTT and STZ models. The simultaneous fit to Eqs. (2.17) and (2.19) yielded the following values of the parameters $a_p=0.037\pm 0.001$, $r_0=0.11\pm 0.001$, and $p_0=0.68\pm 0.08$ with a $\chi=0.709$. The quality of this fit may be observed in Fig. 5. It can be seen that the adjustment to MTT equations is worse than to STZ ones, as suggested by the larger value of χ. The difference at the final stage of the hyperinflation is very important. This feature may be more clearly seen in the case of $r(t)$.

According to our latter results, Zimbabwe is currently suffering the consequences of the *explosion* of its economy. At the beginning of 2009 the highest denomination of the currency was 50×10^9 ZW dollars (this banknote may be seen in [27]). There are reports on very high unemployment and on extreme shortages of basic foodstuffs, fuel, and medical supplies. The number of ill people with severe diseases is increasing, in particular, started an epidemic outbreak of cholera. Many citizens are trying to leave the country. In view of all these facts the government of Zimbabwe must immediately introduce fundamental reforms to avoid the disintegration of the nation due to the complete collapse of the country's economy.

IV. SUMMARY

Hyperinflation episodes in economy were studied in the framework of a model based on adaptive inflation expectations of people. In such a model [5], the growth rate of logarithmic price evolves as a power law with exponent $1/\beta$

toward a spontaneous singularity at a certain t_c. The trend is determined by a collective behavior of all the agents, the effect of other quantities of standard economic theories do not contribute explicitly. Here it is shown that the CPI may diverge as a power law for $0<\beta<1$ or logarithmically for $\beta=1$.

In the present work the expressions for the GR of logarithmic price and the CPI are written in terms of free parameters of the model. In this way one may analyze both the differential observable (GR of logarithmic CPI) and the integrated one (CPI) simultaneously examining the self-consistency of the model. In addition, estimations of uncertainties of the model parameters are provided.

The data of Peru were reanalyzed in order to check our procedures. The self-consistency of the model was satisfactorily verified in all the studied episodes. It is shown that the very extreme historical cases of Greece and Yugoslavia can be well described by the present formalism. Both these examples belong to the worst episodes in the modern history of hyperinflation. The data of the CPI in Greece are consistent with a logarithmic singularity. It is worthwhile to notice that the entire series of measured data in the case of residual Yugoslavia can be reproduced by the present model. The last observations match very well into the general trend toward $P(t=1994:01)\simeq 10^{52}$.

The last analyzed example was the spiral of hyperinflation experienced in Zimbabwe. On the basis of the obtained results, we can state that this country is nowadays suffering an economic crash caused by the unstopped hyperinflation. The government must act immediately to prevent very dreadful consequences for the society. The fact that the critical time for this crash was predicted one year ago [12] indicates that the power of prediction of this model is acceptable.

It is shown that the parameters, including the critical time t_c, are robust to changes of t_0. The fits to STZ and MTT models were compared. It was found that the STZ model provides a better description of the final stage of a hyperinflation than the MTT one.

Let us finish emphasizing that these lessons about the damages of a hyperinflation should not be lost, but instead should be kept in mind to avoid the repetition of that unpleasing experiences. Moreover, one should always remain the Keynes' statement [29]: "even the weakest government can enforce inflation when it can enforce nothing else."

ACKNOWLEDGMENTS

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